Economics Department

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How much to collude without being detected

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Abstract

This paper gives guidelines to antitrust lawyers in a case of alleged tacit collusion on how to report demand and cost parameters such that a collusive outcome becomes indistinguishable from a Cournot outcome. It discusses both joint profit maximising outcomes and less extreme collusive outcomes. In particular, it shows what degree of collusion can be made indistinguishable when the antitrust authority knows that the true parameters must lie below some upper bounds.

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1 Introduction

In the 1996 Papers and Proceedings of the European Economic Review, Phlips' presidential address included a table giving "The Ten Commandments" for the defense to follow in an antitrust proceeding. The assumption is that an antitrust authority (in the following simply termed "authority") has initiated a proceeding against a group of firms, alleging tacit collusion, that is, a collusive outcome sustained as an equilibrium of a noncooperative repeated game. The firms defend themselves by reporting information about demand and cost of production. Since the authority is at an informational disadvantage, the firms under attack can report incorrect values such that the observed prices and production quantities appear as resulting from competitive behaviour. We assume that the authority acknowledges that in a market with a small number of firms "competition" is not equivalent to "price equal marginal cost", but accepts that the firms have positive equilibrium profits as is the case in the noncooperative one-shot equilibrium of the Cournot model of oligopoly.

The "Commandments" suggest that the defense should exaggerate the level of demand, the inelasticity of demand and the marginal cost of production. Suppose that the intercept, the slope (of the inverse demand function) and constant marginal cost are the only three parameters to report. It should be clear that the defense should not blindly follow the Commandments and always over-report all three parameters. There is an economic logic to be followed: the reported parameter values should be compatible with the observed prices and quantities and with the demand and cost functions. There is also an algebraic constraint: the parameter values to report are found by solving a system of only two equations (one defining the quantity and one defining the price), so that there is one degree of freedom.

The purpose of this paper is to clarify this logic (by showing which false reports are compatible with each other) and thus to give clear guidelines to antitrust lawyers who have to decide which defense strategy to follow. Throughout this paper, we will assume that the antitrust authority has no means to verify whether the firms are actually colluding or not, and whether the reported parameter values are the true ones. This means that in our model the authority cannot prove that the firms are colluding, but instead can check whether the reports are consistent with its own information and the price and quantities observed in the market. We assume that the amount of the authority's information is common knowledge.

In the existing literature on this topic it is usually assumed that the authority checks for collusion with a certain probability, and when it does it is always able to identify collusion with certainty. There the demand function is common knowledge, and the cost of production can be exactly determined once the proceeding has started (see for example Besanko and Spulber (1989) and Friedman, Jehiel and Thisse (1995)).

We do not take into consideration the possibility of "cheating" by cartel members, that is deviating from the (tacit) agreement. Rather, we want to delineate how much collusion is possible if cheating can be ruled out. Thus, our results yield upper limits for the possible outcomes that can be reached through repeated play, as in Friedman (1971,1977), where the collusion is enforced by possible future punishment.

In Section 2, we consider the extreme case of a collusive outcome with joint profit maximisation implying the monopoly outcome. We denote this case *full collusion.* This was the assumption adopted in the initial paper by Harstad and Phlips (1994), from which most of the Ten Commandments follow. We find that if the authority does not know any of the parameters, the firms can choose between different possible reports that ensure that no proof of collusion is possible. However, we argue that overreporting all parameters seems to be preferable to overreporting some of the parameters and underreporting the remaining ones. If the authority knows one of the demand parameters, the firms have no choice but to report the remaining demand parameter truthfully and exaggerate the cost of production. If instead the cost is known, both demand parameters must be overstated. In the latter two cases collusion can be made indistinguishable even though the authority has some information. Finally, if the authority knows the cost and any one of the demand parameters, it has conclusive evidence of collusion.

In Section 3, the defense strategies obtained under full joint profit maximisation are reconsidered, allowing for less extreme forms of tacit collusion. Here a market outcome that is intermediate between full collusion and Cournot competitionis is supposed to be given. We call this *partial collusion*. We show that a) the qualitative guidelines (which parameters to over-report in various cases) do not change with the degree of collusion but b) the numerical values to report increase with the degree of collusion.

In Section 4, firms choose the maximum degree of collusion compatible with the fact that the antitrust authority is somewhat better informed and knows that the true parameter values must lie below some upper bounds. If these bounds are large compared to the true values, then the monopoly outcome can be achieved .

2 Reports under Full Collusion.

2.1 The Model

Let the inverse demand function be given by $p = a - bQ$, where p is the market price, a is the demand intercept, b is its slope, which gives a measure of the elasticity of the demand, and Q is the aggregate output of the industry. The form of this inverse demand function is common knowledge. The firms know the parameters a and b, but the authority may not.

We assume that the industry is composed of n identical firms with constant marginal costs c $(c < a)$ and no fixed costs; these firms follow quantity strategies.

Joint profit maximisation leads to $p^J = \frac{a+c}{2}$ and aggregate output $Q^J = \frac{a-c}{2b}$, which assigns to each single firm the quantity $q^J = \frac{a-c}{2bn}$.

On the other hand, in Cournot competition each firm produces $q^c = \frac{a-c}{b(n+1)}$, the aggregate output is $Q^c = n\frac{a-c}{b(n+1)}$ at a price of $p^c = \frac{a+nc}{n+1}$.

Suppose $n = 3$, $a = 14$, $b = 2$, $c = 10$, so that the Cournot price is $p^c = 11$, the monopoly price is $p^J = 12$. Production quantities for each firm are given by $q^c = 0.5$ and $q^J = 0.33$. Aggregate quantities are $Q^c = 1.5$ and $Q^J = 1$. We will use this example throughout the paper to illustrate our results.

The antitrust authority wants the firms to compete which we interpret as engaging in Cournot competition. If the authority suspects that the firms are colluding, it will ask them to report demand and cost parameters to check whether the market price is too high and the quantities are too small as one would find under collusion. If the firms are actually colluding they will want to report demand parameters $\widehat{a}$, $\widehat{b}$ and marginal cost $\widehat{c}$ such that the price p^J and the quantities q^J seem to result from Cournot competition:

$$p^J = \frac{\widehat{a} + n\widehat{c}}{n+1} \tag{1}$$

$$q^J = \frac{\widehat{a} - \widehat{c}}{\widehat{b}(n+1)}. \tag{2}$$

The following analysis demonstrates what the firms should report in the cases where the authority has different amounts of information about the parameters.

2.2 The Authority has no Information

In this case, given conditions (1) and (2), and the three parameters to report, the problem amounts to solving a system of two equations in three unknowns, which leaves the firm with one degree of freedom. In other words, one parameter value can be chosen freely and the other two must then be calculated using that value. So the values to report are (for proofs see the appendix): Choose $\widehat{a}$ and

$$\widehat{b} = b\frac{\widehat{a} - p^J}{a - p^J} = b\left(1 + 2\frac{\widehat{a} - a}{a - c}\right) \tag{3}$$

$$\widehat{c} = \frac{(n+1)p^J - \widehat{a}}{n} = c + \frac{(n-1)(a-c) + 2(a-\widehat{a})}{2n} \tag{4}$$

where $\widehat{a}$ is chosen freely. Given the value of a, there are three possibilities:

- $\widehat{a} > a$: Here b must always be overstated and c as well if the reported $\widehat{a}$ is not too large:

$$\widehat{b} > b \text{ and } \widehat{c} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} c \text{ if } \widehat{a} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} a + (n-1)\tfrac{a-c}{2} \qquad \begin{array}{c} \text{(i)} \\ \text{(ii)} \\ \text{(iii)} \end{array}$$

- $\widehat{a} = a$: $\qquad \widehat{b} = b$ and $\widehat{c} > c$ (iv)

- $\widehat{a} < a$: $\qquad \widehat{b} < b$ and $\widehat{c} > c$ (v)

Possible reports sustaining the monopoly price and quantity in these five cases are:

	$\widehat{a}$	$\widehat{b}$	$\widehat{c}$
(i)	16	4	10.67
(ii)	18	6	10
(iii)	20	8	9.33
(iv)	14	2	11.33
(v)	13	1	11.67

Although all these five possibilities lead to the desired indistinguishability, the choices (iii) and (v) are not to be recommended: If (v) is compared to (iv), we see that in (v) the defense must overreport cost more than in (iv) to compensate for the underreported demand parameters. In (iii), the defense has to underreport cost to compensate for the exaggerated report of the demand intercept. The intuition is that in general it is advisable not to report values that are too extreme, because they will be more likely to raise suspicions that exaggerated parameter values are being reported. Moreover, underreporting of parameters today will constrain the firms' ability to report higher values in the future.

2.3 The Authority knows the Marginal Cost

The values to report are:

$$\widehat{a} = a + (n-1)\frac{a-c}{2}$$

$$\widehat{b} = nb \qquad\qquad \widehat{c} = c.$$

Clearly, the defense has to overstate a and b. From the above example we have $\widehat{a} = 18$, $\widehat{b} = 6$ and $\widehat{c} = c = 10$.

2.4 The Authority knows at least one Demand Parameter

The values to report are:

$$\widehat{a} = a \qquad\qquad \widehat{b} = b$$

$$\widehat{c} = \frac{(n-1)a + (n+1)c}{2n}.$$

If the authority knows one demand parameter the firms have to report the other one truthfully! But since they are charging the monopoly price and not the Cournot price, some parameter must be overstated, and in this case the only one left is cost $\widehat{c}$. This means that the antitrust authority will not be able to detect collusion if it does not know anything about production cost. From the above example we have $\widehat{a} = a = 14$, $\widehat{b} = b = 2$ and $\widehat{c} = 11.33$.

2.5 The Authority knows the Marginal Cost and one Demand Parameter

The defense has to report the true value for the remaining unknown parameter:

$$\widehat{a} = a \qquad \widehat{b} = b \qquad \widehat{c} = c.$$

There is no point in reporting the unknown demand parameter incorrectly, since the antitrust authority can compute its true value using the value of the known demand parameter. In this case no undetectable collusion is possible, i.e. there is no indistinguishability: tacit collusion can easily be detected by comparing the observed prices and quantities to the values they should obtain in a competitive Cournot equilibrium. As shown above, the authority can only compute these values if it knows the true values of *all* parameters. In our example, $\widehat{a} = a = 14$, $\widehat{b} = b = 2$ and $\widehat{c} = c = 10$.

3 Reports under Partial Collusion

In the previous section we considered the full joint profit maximising outcome. Now we relax this assumption, allowing firms to collude to a lesser degree that is considered to be given (we will look at *choosing* the degree of collusion in the next section). Indeed, the authority may be better informed than assumed up to this point, and then full collusion may not be possible. This section sets the stage for Section 4, where we assume that the authority knows some upper bounds that the true parameter values cannot exceed. Obviously, in most of the following cases full collusion would be optimal; but the aim of this analysis is to derive the formulas needed in Section 4.

We use a rough measure of collusion by indexing the level of prices using a parameter $\beta \in [0,1]$. For given β, the resulting price p is given by

$$p = (1-\beta)p^c + \beta p^J, \tag{5}$$

i.e. p lies between the Cournot price and the monopoly price. Note that

$$\beta = 0 \Longleftrightarrow p = p^c \quad (6)$$

$$\beta = 1 \Longleftrightarrow p = p^J. \quad (7)$$

For our numerical example we get $p = (1-\beta)(11) + \beta(12) = \beta + 11$ and $q = (1-\beta)(0.5) + \beta(0.33) = 0.5 - 0.167\beta$. In the following we will use $\beta = 0.5$, leading to $p = 11.5$ and $q = .417$.

For a given price p we can compute the degree of collusion β from

$$\beta = \frac{p - p^c}{p^J - p^c} \quad (8)$$

Note also that since $p = a - bnq$ is linear in p and q, for $p = (1-\beta)p^c + \beta p^J$ we can derive the quantity that is produced by each firm as

$$q = (1-\beta)q^c + \beta q^J, \quad (9)$$

so that the degree of collusion given by the value of β does not depend on whether we measure collusion in terms of prices or quantities. We now give the results for the cases that are analogous to the previous section.

3.1 The Authority has no Information

If the authority does not know a single true value and the price is given by $p = (1-\beta)p^c + \beta p^J$ for some fixed value of β, then the firms should report some $\widehat{a} \geq a$ and

$$\widehat{b} = b\frac{\widehat{a} - p}{a - p} = b\left(1 + \frac{2(n+1)(\widehat{a} - a)}{[2n - \beta(n-1)](a-c)}\right), \quad (10)$$

$$\widehat{c} = \frac{(n+1)p - \widehat{a}}{n} = c + \frac{2(a - \widehat{a}) + \beta(n-1)(a-c)}{2n}. \quad (11)$$

The demand slope $\widehat{b}$ should be exaggerated, and also the cost report $\widehat{c}$ if $\widehat{a}$ is not too large (the critical value is $\widehat{a} = a + \beta(n-1)\frac{a-c}{2}$). It is straightforward to verify that the cases (i) to (v) from above hold again. Possible reports are $\widehat{a} = 15$, $\widehat{b} = 2.8$, $\widehat{c} = 10.33$

3.2 The Authority knows the Marginal Cost

The values to report are $\widehat{c} = c$ and

$$\widehat{a} = a + \beta(n-1)\frac{a-c}{2}, \tag{12}$$

$$\widehat{b} = nb\frac{2+\beta(n-1)}{2n-\beta(n-1)} = b\frac{2n-\beta n+\beta n^2}{2n-\beta n+\beta}. \tag{13}$$

It is clear that the firms have to overstate both demand parameters if they are colluding. Reporting $\widehat{a} = 16$, $\widehat{b} = 3.6$, $\widehat{c} = c = 10$, ensures indistinguishability.

3.3 The Authority knows at least one Demand Parameter

The values to report are:

$$\widehat{a} = a \qquad \widehat{b} = b, \tag{14}$$

$$\widehat{c} = c + \frac{\beta(n-1)(a-c)}{2n}. \tag{15}$$

As before, if the authority knows one demand parameter the firms have to report the other one truthfully. Cost has to be overstated to obtain indistinguishability. The defense should report $\widehat{a} = a = 14$, $\widehat{b} = b = 2$, $\widehat{c} = 10.67$.

3.4 The Authority knows the Marginal Cost and one Demand Parameter

The defense must report the true value for the remaining unknown demand parameter:

$$\widehat{a} = a \qquad \widehat{b} = b \qquad \widehat{c} = c. \tag{16}$$

This result is rather strong.Even partial collusion, however small, cannot be hidden by false reports! The firms have no other choice but to engage in Cournot competition, and counsel should realise that pleading "not guilty" is hopeless.

4 Maximum Indistinguishable Collusion

4.1 The Firms' Problem

In this section we will assume that the authority knows that the true values of the demand and cost parameters lie below some fixed upper bounds, and that this fact and the values of the bounds are common knowledge. It is clear that the reported parameter values should not exceed these bounds, since such a report would be an obvious sign of collusion. On the other hand, if the bounds are close to the true values, full joint profit maximization will no longer be feasible, and only some degree of partial collusion can be achieved.

We are ignoring lower bounds since we concluded in Section 2 that there is no incentive for the firms to underreport any of the parameters. The firms' reports then have to satisfy the following bounds:

$$\widehat{a} \leq a_H \qquad \widehat{b} \leq b_H \qquad \widehat{c} \leq c_H, \tag{17}$$

where the subscript H stands for upper bound. We assume that the authority's bounds are never smaller than the true values of a, b, and c. Note that the analysis of the preceding sections can be incorporated in this setting by assuming that if the authority actually knows the value of some parameter, the "upper bound" is the true value of that parameter. If for example the authority knows a, this is equivalent to imposing the upper bound $a_H = a$.

The firms face the following optimisation problem: find the degree of collusion that maximises individual profits and ensures indistinguishability conditional on the given bounds. At the optimum firms choose the price given by (see the appendix for the proof):

$$p = \min\{p^J = \frac{a+c}{2}, p^a = \frac{a_H + nc_H}{n+1}, p^b = \frac{b_H a + nbc_H}{b_H + nb}\}. \tag{18}$$

The optimal price is the highest price that is *feasible*, which means that the firms have to choose the smallest one out of $\{p^J, p^a, p^b\}$. This is because, given the bounds, the firms may not be able to collude as much as they want to. The choice of price depends on whether the authority's bounds are so high that they are not affecting collusion, or whether they are so low that the firms cannot report arbitrarily high values.

- Case 1: $p = p^J$: This is the monopoly price and thus the price chosen by the firms under joint profit maximisation if the bounds given by the authority are very high. In this respect, this case clearly coincides with the case without bounds. Still, the reported $\widehat{a}$ will have to be in a range given by

$$\max\{a, (n+1)\frac{a+c}{2} - nc_H\} \leq \widehat{a} \leq \min\{a_H, a + (\frac{b_H}{b} - 1)\frac{a-c}{2}\}, \tag{19}$$

  since the choice of values is restricted (see the figure for case 1). The reports $\widehat{b}$ and $\widehat{c}$ are computed from the chosen $\widehat{a}$ using the formulas from above:

$$\widehat{b} = b\frac{\widehat{a} - p^J}{a - p^J} \qquad \widehat{c} = \frac{(n+1)p^J - \widehat{a}}{n}.$$

  Continuing our numerical example, suppose that $a_H = 15$, $b_H = 3$, and $c_H = 12$. We obtain $p^J = 12$, $p^a = 12.75$ and $p^b = 12.67$, such that the firms will be able to charge the monopoly price.

- Case 2: $p = p^a$: This price will be charged when counsel cannot do any better than report values for a and c that coincide with the upper bounds. This means that the constraints on $\widehat{a}$ and $\widehat{c}$ are binding whereas the constraint on $\widehat{b}$ is not (see the figure for case 2). The firms should report:

$$\widehat{a} = a_H, \qquad \widehat{c} = c_H, \tag{20}$$

$$\widehat{b} = b\frac{n(a_H - c_H)}{(n+1)a - a_H - nc_H} = b\frac{(n+1)a_H - a_H - nc_H}{(n+1)a - a_H - nc_H} \tag{21}$$

where the last expression makes it clear that $\widehat{b} > b$ if $a_H > a$.

If we assume that $a_H = 15$, $b_H = 3$, and $c_H = 10$, we obtain $p^J = 12$, $p^a = 11.25$ and $p^b = 11.33$. In this case the low bounds on cost (this bound is actually the true value) and the demand intercept prevent the firms from colluding fully. Compare this with Section 2.3 and note that the bound on the demand slope, though also low, does not matter since the bound on the demand intercept is "tighter".

- Case 3: $p = p^b$: This price will be charged when at the optimum the constraints on $\widehat{b}$ and $\widehat{c}$ are binding whereas the constraint on $\widehat{a}$ is not (see the figure for case 3). The firms will report:

$$\widehat{a} = \frac{(n+1)ab_H - n(b_H - b)c_H}{b_H + nb} = a + \frac{n(b_H - b)(a - c_H)}{b_H + nb}, \tag{22}$$

$$\widehat{b} = b_H, \qquad \widehat{c} = c_H. \tag{23}$$

The reported $\widehat{a}$ will be larger than a.

Suppose that $a_H = 15$, $b_H = 2$, and $c_H = 11$, we obtain $p^J = 12$, $p^a = 12$ and $p^b = 11.75$. Here the firms will only be able to charge p^b.

4.2 The Maximum Degree of Collusion

Now we can compute the maximum degree of collusion that is indistinguishable from non-cooperative behaviour. To do this we take into account the upper bounds on the reported parameters and the resulting optimal prices. This will provide us with a measure of how high collusion is in every case. Remember that the degree of collusion was defined by

$$\beta = \frac{p - p^c}{p^J - p^c}.$$

Thus, substituting the optimal prices from (18) we get, respectively:

Case 1: $p = p^J = \frac{a+c}{2}$:

$$\beta = 1. \tag{24}$$

Of course, if the firms charge the monopoly price we have full collusion.

Case 2: $p = p^a = \frac{a_H + nc_H}{n+1}$:

$$\beta = 2\frac{(a_H - a) + n(c_H - c)}{(n-1)(a-c)}. \tag{25}$$

In our example, we obtain $\beta = 0.25$.

Case 3: $p = p^b = \frac{b_H a + nbc_H}{b_H + nb}$:

$$\beta = 2n\frac{(b_H - b)a + nb(c_H - c) + (bc_H - b_H c)}{(n-1)(a-c)(b_H + nb)}. \tag{26}$$

Here the example leads to $\beta = 0.75$.

The interpretation of these values is that in cases 2 and 3 only partial collusion can be made indistinguishable. We can see from expressions (25) and (26) that the closer the bounds are to the true values, the smaller the resulting degree of collusion will be.

A Joint Profit Maximisation under a given Degree of Collusion

In this appendix we derive the reports ensuring indistinguishability for arbitrary $\beta \in [0, 1]$. The case of full profit maximisation at the monopoly price level is obtained by setting $\beta = 1$. First we derive the general formulas where the authority knows nothing and then specialize them to the cases where the authority has some information about true values of cost and/or demand parameters.

For a fixed degree of collusion β the firms choose a price p between the monopoly and the Cournot price, and determine the individual quantities q from the demand function:

$$\begin{aligned} p = (1-\beta)p^c + \beta p^J &= (1-\beta)\frac{a+nc}{n+1} + \beta\frac{a+c}{2}, \qquad (27) \\ q &= \frac{a-p}{nb}. \end{aligned}$$

To insure indistinguishability, the firms must report a set of parameter values $(\widehat{a}, \widehat{b}, \widehat{c})$ such that p and q appear as Cournot price and quantity, respectively:

$$p = \frac{\widehat{a} + n\widehat{c}}{n+1} \qquad\qquad q = \frac{\widehat{a} - \widehat{c}}{\widehat{b}(n+1)}. \qquad (28)$$

Since (28) is a system of two equations in three unknowns $(\widehat{a}, \widehat{b}, \widehat{c})$ we can choose an arbitrary value for $\widehat{a}$ and solve for $\widehat{b}$ and $\widehat{c}$:

$$\widehat{b} = b\frac{\widehat{a}-p}{a-p} = b\left(1 + \frac{2(n+1)(\widehat{a}-a)}{[2n-\beta(n-1)](a-c)}\right), \qquad (29)$$

$$\widehat{c} = \frac{(n+1)p - \widehat{a}}{n} = c + \frac{2(a-\widehat{a}) + \beta(n-1)(a-c)}{2n}. \qquad (30)$$

From (29) we see that the relationship between $\widehat{a}$ and $\widehat{b}$ is monotonically increasing. Note that $\widehat{b} = b$ if and only if $\widehat{a} = a$: If the authority knows one demand parameter the other one has to be reported truthfully; in this case $\widehat{c} = c + \frac{\beta(n-1)(a-c)}{2n}$ from (30). On the other hand, if c is known, i.e. $\widehat{c} = c$, then again from (30) we see that $\widehat{a} = a + \beta(n-1)\frac{(a-c)}{2}$ and, using this result and (29), $\widehat{b} = nb\frac{2+\beta(n-1)}{2n-\beta(n-1)}$. □

B Choosing the maximum degree of collusion

Here we calculate the profit-maximising degree of collusion that the firms can obtain if it is common knowledge that the demand and cost parameters lie below some fixed upper bounds. Then the firms have to solve the following optimisation problem:

Choose $(p, q, \widehat{a}, \widehat{b}, \widehat{c})$ to maximise individual profits $\pi = (p - c)q$ given that price p and quantity q satisfy the demand equation, and p lies between the Cournot and the monopoly price:

$$p = a - bnq, \qquad\qquad p^c \leq p \leq p^J. \tag{31}$$

The reports $(\widehat{a}, \widehat{b}, \widehat{c})$ must be such that p and q appear as resulting from Cournot competition:

$$p = \frac{\widehat{a} + n\widehat{c}}{n+1} \qquad\qquad q = \frac{\widehat{a} - \widehat{c}}{(n+1)\widehat{b}}, \tag{32}$$

and must lie within the bounds:

$$a \leq \widehat{a} \leq a_H \qquad\qquad b \leq \widehat{b} \leq b_H \qquad\qquad c \leq \widehat{c} \leq c_H. \tag{33}$$

Using equations (31) and (32) we can substitute for q, $\widehat{b}$ and $\widehat{c}$ using

$$q = \frac{a - p}{nb} \quad \widehat{b} = b\frac{\widehat{a} - p}{a - p} \quad \widehat{c} = \frac{n+1}{n}p - \frac{1}{n}\widehat{a}, \tag{34}$$

thus eliminating these variables from the problem. Then, profits can be rewritten as

$$\pi = (p - c)\frac{a - p}{nb} = \frac{(a + c)p - p^2 - ac}{nb}.$$

We see that

$$\frac{\partial \pi}{\partial p} = \frac{a+c-2p}{nb} > 0 \text{ if } p < p^J = \frac{a+c}{2}.$$

Since profits are increasing in p as long as it is below the monopoly price, profit maximisation is equivalent to choosing the maximum price given the constraints (31), (32) and (33). Using equation (34), we can restate the firms' problem as the following Linear Programming problem:

$$\max_{p,\widehat{a}} p$$

s.t.

$$\frac{a+nc}{n+1} \leq p \leq \frac{a+c}{2} \tag{35}$$

$$a \leq \widehat{a} \leq a_H \tag{36}$$

$$\widehat{a} + \left(\frac{b_H}{b} - 1\right) p \leq \frac{b_H}{b} a \tag{37}$$

$$-\widehat{a} + (n+1)p \leq nc_H. \tag{38}$$

If at the optimum (35) is binding then $p = p^J$, and the bounds on $\widehat{a}$ can be derived from (36), (37), and (38) with $p = p^J$ (see figure illustrating case 1 in the text).

If (35) is not binding, then the firms cannot reach full collusion since some constraints on $\widehat{a}$, $\widehat{b}$, or $\widehat{c}$ must be binding. In this case, the constraint (38) on $\widehat{c}$ must be binding. Assume it does not: Since (35) is not binding we have $p < p^J$, and (at least) one of (36), (37) is binding. Obviously, p is then maximized for $\widehat{a} = a$, yielding $p = a$ which contradicts $p < p^J < a$.

So assume that (38) is binding: $\widehat{c} = c_H$. If in addition (36) is binding, then $p = p^a = \frac{a_H + nc_H}{n+1}$ and $\widehat{a} = a_H$, $\widehat{b} = b\frac{n(a_H - c_H)}{(n+1)a - a_H - nc_H}$, $\widehat{c} = c_H$ by solving (38) and (36) as equalities (see figure for case 2). If instead (37) is binding, we get $p = p^b = \frac{b_H a + nbc_H}{b_H + nb}$, $\widehat{a} = \frac{(n+1)ab_H - n(b_H - b)c_H}{b_H + nb}$,$\widehat{b} = b_H$, $\widehat{c} = c_H$, respectively (see figure for case 3). □

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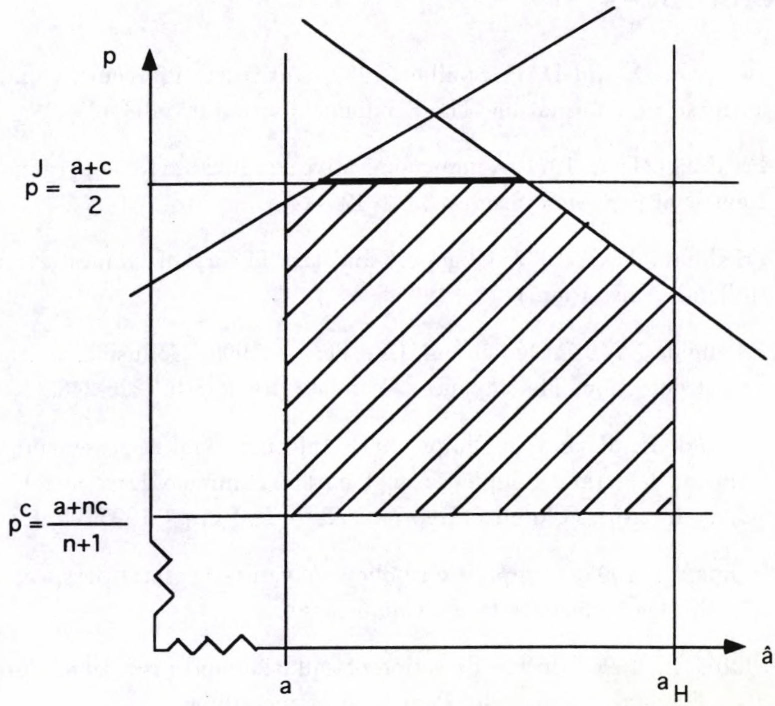
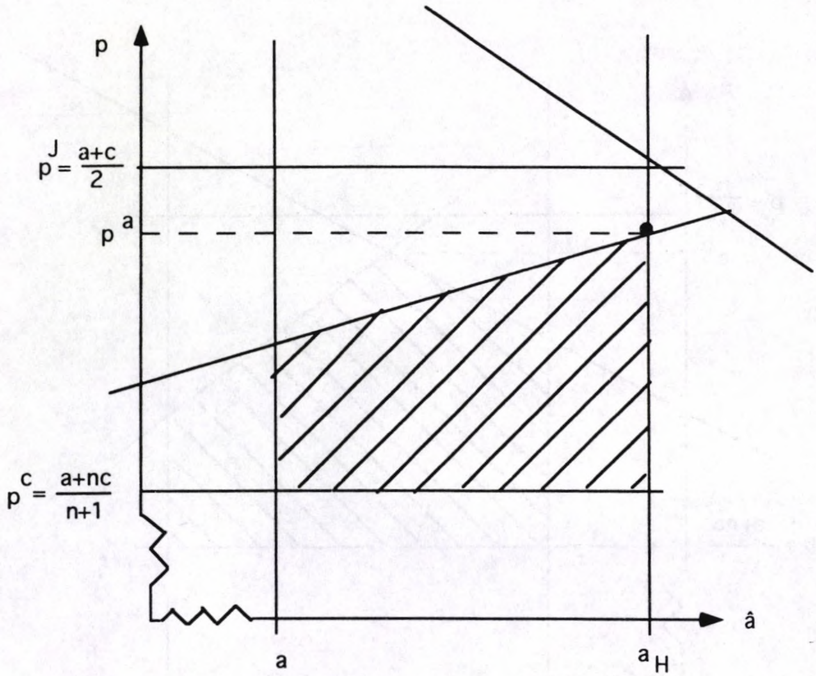


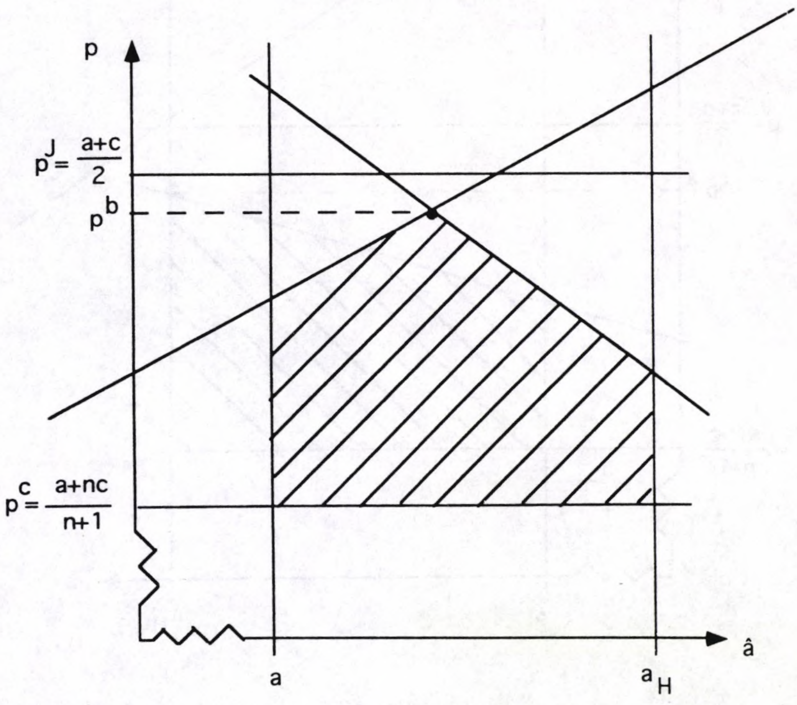
<u>Case 1</u>

Reported parameter values are smaller than the bounds



Case 2

The bounds on the intercept
and the cost reports are binding



<u>Case 3</u>

The bounds on the slope and
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